Use of Positively Biased Electrostatic Probes to Obtain Electron Density in Collisionless Flows

MICHAEL G. DUNN*
Cornell Aeronautical Laboratory, Buffalo, N. Y.

Voltage-swept thin-wire electrostatic probes have been used to obtain the electron density in the freestream flow of a reflected-shock tunnel and in the boundary layer over a flat plate located in this expanding-flow environment. The voltage sweep was sufficient to permit electron-density determination from both the ion- and electron-current portions of the probe characteristic while the electron temperature was determined from the electron-retarding region. The electron density determined from these two portions of the probe characteristic were in good agreement with each other and with independent microwave measurements for the freestream experiments. Laframboise's free-molecular flow theory was used to deduce the number-density results presented here.

Nomenclature

e = electron charge
i_+, i_- = normalized ion and electron current densities
j = current per unit area of probe surface
k = Boltzmann's constant
m = particle mass
n_e = number density of electrons
R_p = probe radius
T = temperature
V = voltage
V_f = voltage at floating potential
V_∞ = voltage at plasma potential
λ_D = Debye length
λ = mean free path
χ_p = dimensionless potential difference between probe and undisturbed plasma

Subscripts

e = electron
i = ion
∞ = undisturbed plasma

Introduction

THERE are several well-known[1–5] advantages of using the electron-current portion of an electrostatic-probe current-voltage characteristic for determination of electron density. Perhaps the most important advantages are that the collected current is independent of the ion mass and also independent of the orientation relative to the mass motion of heavy particles because of the relatively high electron thermal speed. In addition, the collision cross sections for electron-particle interactions are generally smaller than those for ion-particle interactions suggesting that the collisionless-flow approximation should be valid over a greater range of gas densities. Further, the electron current collected by the probe can be made insensitive to the electron temperature by working at large probe voltages. Sutton[6] has described the feasibility of using thin-wire probes biased for electron current in order to measure electron-density fluctuations in hypersonic turbulent wakes. However, relatively few[5] experimental studies have been reported in the literature that have attempted to show that the electron-current region can be successfully used to obtain these electron-density measurements within the framework of existing theoretical results. It is thus the purpose of this paper to report the results of an experimental study undertaken in order to determine if the existing theoretical results of Laframboise[7] for the current collected by thin-wire probes operating in the free-molecular flow regime could be successfully used in the electron-current region. Many people have illustrated the correctness of this theory for the ion-current region. The details of Laframboise's work are well-known and will not be described here.

The current studies were motivated by the availability of inflight voltage-swept probe data obtained by Jones[8] as part of the Langley Research Center RAM program. The voltage applied to his probes was sufficiently positive that the probes were swept well into the electron-current region at altitudes for which the free-molecular flow approximation should be valid. Electron densities have been obtained from the ion-current portion of his probe characteristics and are reported elsewhere.[8]

Center[5] used a pressure-driven shock tube to produce a plasma behind the incident shock in which he performed probe experiments under free-molecular flow conditions. The voltage on his probes was held constant during the experiment because of the rapidly varying conditions behind the incident shock. By performing a series of experiments he was able to construct the current-voltage characteristic and subsequently show that the experimental data are predicted by the theoretical results of Laframboise.[7] The results presented in this paper are in agreement with those described by Center. However, the fluid-flow environments in which the experiments were performed were significantly different and the probe characteristics were obtained by sweeping the probe voltage instead of maintaining constant voltage.

Experimental Apparatus and Technique

A pressure-driven shock tube was used to produce a reservoir of high-temperature gas which was subsequently expanded in a conical nozzle constructed of Fiberglas. A detailed discussion of this shock tube and nozzle is given in Refs. 9 and 10.

The majority of the probe measurements reported here were performed in the boundary layer of a sharp flat plate mounted in the nozzle flow such that its leading edge was 22.5 in. from the throat and its top surface was on the nozzle centerline. A few experiments were also performed in the absence of the plate with the probes located on the nozzle centerline at 32.5 in. from the throat. An independent and simultaneous measurement of averaged freestream electron density was obtained at 11.5 and 21.5 in. (31.5 in. for the freestream experiments) from the throat using microwave interferometers operating at 35 and 17 GHz, respectively. Carbon monoxide was used as the test gas for the boundary-layer experiments. The carbon monoxide test gas was expanded from an equilibrium reservoir condition of 7060°K at 17.3 atm pressure. However, the freestream measurements were performed in both carbon monoxide and nitrogen test gases. The nitrogen test gas was expanded from an equilibrium

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* Principal Engineer, Aerodynamic Research Department. Member AIAA.

reservoir condition of 7200°K at 17.1 atm pressure. Previous boundary-layer experiments have been reported[11] which used nitrogen as the test gas but in these measurements the probes were not swept into the electron-current region.

The carbon monoxide used here was ultra high-purity grade supplied by Air Products and Chemicals Inc. The principal impurity in this gas was nitrogen estimated to be on the order of 1500 to 2000 ppm. The nitrogen test gas was ultra-pure carrier grade also supplied by Air Products and Chemicals Inc. A chemical analysis of the nitrogen gas indicated the following: oxygen less than 0.5 ppm, total hydrocarbons less than 1 ppm, and water less than 0.15 ppm.

The probes used in these experiments were constructed by surrounding 0.004-in.-diam tungsten wires with a quartz envelope, leaving a nominal 0.400 in. length of bare wire exposed. Immediately prior to each run, the tungsten oxide is removed by placing the probe in a dilute solution of sodium hydroxide and passing approximately 400 μa of current through the circuit for approximately 10 min.

Discussion of Results

By using Laframboise's[7] theory to obtain electron temperatures and electron densities from the experimentally determined probe characteristics, it has been assumed that the flow conditions were collisionless with respect to the probe diameter. At all of the measuring stations, the electron mean free paths were many times greater than the probe diameter. Because of the relative insensitivity of electron collection to the magnitude of the ion-neutral and neutral-neutral mean free paths, it is anticipated that the electron temperature deduced from the electron-retarding region and the electron density deduced from the electron-current region of the probe characteristic should be correct. The ion-neutral mean free path was the shortest, being slightly greater than the probe diameter at the upstream measuring station. However, the electron-density results presented later in this paper suggest that the ion-neutral mean free path apparently had a relatively insignificant influence on the collected ion current. The calculated mean free paths and species concentrations in the expanding carbon monoxide and nitrogen flows used for the present experimental test conditions are given in detail in Refs. 12 and 13, respectively.

It is important to note that the carbon monoxide test gas used in the work reported in Ref. 12 had substantially less impurities (25 ppm nitrogen) than that used in the present experiments (1500 to 2000 ppm nitrogen). It was not possible to obtain additional CO from Lif-O-Gen Inc. with the low nitrogen content. The additional nitrogen appeared to influence the chemical kinetics of the expanding plasma by decreasing the electron-density level by a factor of approximately two throughout the expansion. For the purposes of this study the details of the kinetics are important and had to be considered. Even with the nitrogen present, the dominant ion in the expansion was calculated to be C^+ so that ion mass 12 was used in the analysis of the ion-current data. The method of probe-data reduction used here has been outlined in detail in Ref. 14 and will not be repeated. The familiar normalized ion- and electron-current densities (i_+ and i_-) were taken from Laframboise's theoretical results for $T_i/T_e = 0$ and are given in Figs. 1 and 2, respectively. For the experimental conditions considered here the $T_i/T_e = 0$ theoretical results are satisfactory since the freestream ion temperature to electron temperature ratio was on the order of 0.1 to 0.2.

Table 1 presents a summary of the experimental results obtained as part of this study. Included in this table are the electron

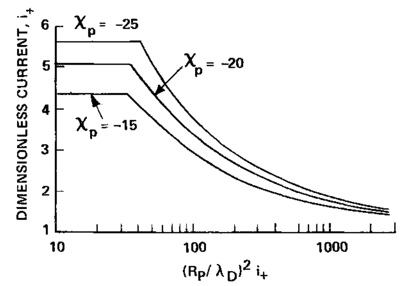

Fig. 1 Laframboise theoretical result for ion-current collection when $T_i/T_e = 0$.

Table 1 Results obtained from langmuir probe current-voltage characteristics

Test gas	Probe location	μ-wave number density at 31.5 in. e^-/cm^3	Number density from ion current e^-/cm^3	R_p/λ_D	T_e, °K	Number density from electron current e^-/cm^3	R_p/λ_{D-}	$(V_\infty - V_f)_{est}$, v	$(V_\infty - V_f)_{exp}$, v
CO	freestream	2.0×10^{11}	1.5×10^{11}	7.5	2110	1.3×10^{11}	6.9	0.9	0.8
CO		1.5×10^{11}	1.7×10^{11}	8.8	1650	1.7×10^{11}	8.7	0.7	0.9
CO		2.0×10^{11}	1.2×10^{11}	7.0	1945	1.4×10^{11}	7.4	0.85	0.8
CO	boundary layer		1.4×10^{10}	1.7	2500	1.3×10^{10}	1.7	1.2	1.2
CO			1.3×10^{10}	1.9	2090	1.3×10^{10}	1.8	0.9	1.1
CO			1.7×10^{10}	2.0	2320	1.4×10^{10}	1.4	1.0	1.1
CO			4.4×10^{10}	3.3	2300	3.3×10^{10}	2.8	1.0	1.2
CO			1.6×10^{10}	1.9	2510	1.4×10^{10}	1.8	1.1	0.9
CO			6.6×10^{10}	4.3	2000	5.3×10^{10}	3.8	0.9	0.9
CO			1.1×10^{10}	1.6	2170	1.1×10^{10}	1.7	0.95	0.9
CO			5.3×10^{10}	3.5	2310	5.0×10^{10}	3.4	1.0	1.2
CO			1.8×10^{10}	2.3	1810	1.8×10^{10}	2.3	0.8	0.8
CO			6.2×10^{10}	4.4	1730	6.5×10^{10}	3.6	0.76	0.8
CO			2.1×10^{10}	2.3	2100	2.1×10^{10}	2.3	0.92	1.0
CO			4.1×10^{10}	3.4	1950	4.1×10^{10}	3.4	0.85	0.9
CO			8.1×10^{9}	1.5	1955	8.9×10^{9}	1.6	0.86	0.8
CO			5.0×10^{10}	3.9	1960	4.6×10^{10}	3.6	0.86	0.9
CO			2.5×10^{10}	2.5	2130	2.6×10^{10}	2.5	0.93	1.1
CO			9.4×10^{9}	1.6	2020	9.2×10^{9}	1.6	0.88	0.8
CO			2.7×10^{10}	2.7	2060	2.9×10^{10}	2.8	0.90	1.0
CO			1.2×10^{10}	1.7	2160	1.3×10^{10}	1.8	0.93	0.9
CO			2.8×10^{10}	2.8	1980	3.8×10^{10}	3.2	0.87	1.0
CO			6.8×10^{9}	1.3	2160	6.5×10^{9}	1.3	0.95	1.1
CO			4.3×10^{10}	3.5	1915	4.3×10^{10}	1.4	0.84	1.0
N_2	freestream	9.5×10^{9}	9.0×10^{9}	1.4	3580	1.1×10^{10}	1.5	1.6	2.0
N_2		1.0×10^{10}	1.1×10^{10}	1.5	3670	1.1×10^{10}	1.5	1.6	1.6
N_2		1.5×10^{10}	1.4×10^{10}	1.7	3510	1.1×10^{10}	1.5	1.53	1.5
N_2		7.2×10^{9}	7.4×10^{9}	1.5	2450	3.5×10^{9}	2.1	1.06	0.7

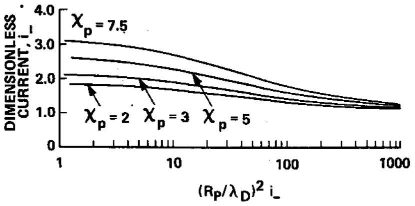

Fig. 2 Laframboise theoretical result for electron-current collection when $T_i/T_e = 0$.

densities determined from the ion- and electron-current portions of the current-voltage characteristics, the calculated values of R_p/λ_D for these regimes, a comparison of $(V_\infty - V_f)$ obtained from the tangent intercept technique and an estimate obtained from the theory, the electron temperatures determined from the electron-retarding region, and electron densities determined using a microwave interferometer when freestream probe measurements are given.

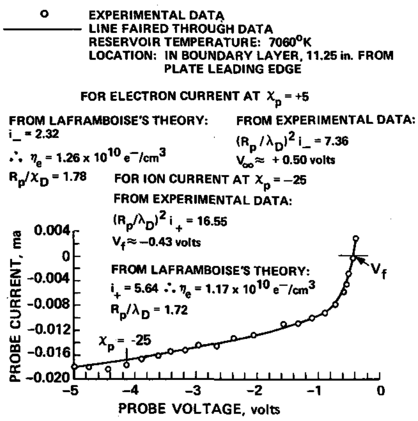

Fig. 3 Ion-current region of experimental probe characteristic in carbon monoxide.

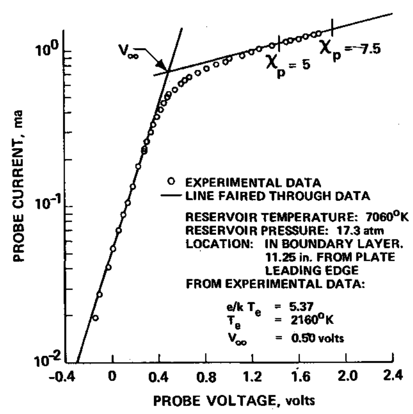

Fig. 4 Electron-retarding and electron-current regions of experimental probe characteristic in carbon monoxide.

Figures 3 and 4 present the ion-current region and the electron-retarding and electron-current regions respectively of the probe characteristic obtained in the boundary layer for a typical experiment. On Fig. 3 the ion current is utilized to deduce the number density at $\chi_p = -25$. For this particular case the value of $(R_p/\lambda_D)^2 i_+$ was 16.55 which is in the orbital-motion limited regime. The corresponding value of R_p/λ_D was 1.72 and the floating potential was -0.43 v. The electron density calculated using the theoretical results of Laframboise[7] was 1.2×10^{10} e^-/cm^3. Also shown on Fig. 3 is the electron density calculated using the electron current at a potential of $\chi_p = +5$. The electron current at $\chi_p = +7.5$ was also used but the resulting electron density was essentially unchanged. The value of $(R_p/\lambda_D)^2 i_-$ at $\chi_p = +5$ was calculated to be 7.36 and the corresponding value of R_p/λ_D was 1.78 which is reasonably close to the value calculated from the ion current. The number density deduced from the electron current was $1.3 \times 10^{10} e^-/\text{cm}^3$ which is very good agreement with the value previously obtained from the ion-current portion of the characteristic.

The electron temperature was obtained from the results presented in Fig. 4. The value of e/kT_e for this particular experiment was 5.37 or $T_e = 2160°$K. By extrapolating the line faired through the retarding-field region to intersect with a line faired through the electron-current region one can obtain an estimate of the plasma potential. For this case, the extrapolation gave a plasma potential of $+0.5$ v; the floating potential, which is easily determined from Fig. 3, was -0.43 v resulting in an esperimental value of $(V_\infty - V_f) = 0.93$ v. Using the theoretical expressions given in Eqs. (1) and (2) of Ref. 3 and equating j_i to j_e at the floating potential, one can obtain an estimate of the value of $(V_\infty - V_f)$ from the theory to be given by $(kT_e/e)\,\text{Ln}(m_i/m_e)^{0.5}\, i_+$. We will refer to this value as $(V_\infty - V_f)_{\text{est.}}$ and compare it in Table 1 with $(V_\infty - V_f)_{\text{exp.}}$ determined from the experimental data using the tangent intercept method in a manner illustrated in Fig. 4. For the purposes of this paper, i_+ was assumed to be equal to 1.0 which results in a maximum error in $(V_\infty - V_f)$ of 10 to 12% for the conditions of these experiments.

Assuming C^+ to be the dominant ion, then $(V_\infty - V_f)_{\text{est.}} \cong (T_e/2320)$; then assuming N^+ to be the dominant ion in the nitrogen experiments, gives $(V_\infty - V_f)_{\text{est.}} \cong T_e/2290$. Using the previous expression for the C^+ ion the value of $(V_\infty - V_f)_{\text{est.}}$ for the results presented in Figs. 3 and 4 is equal to 0.932 as compared to the experimental value of 0.93. The agreement between $(V_\infty - V_f)$ estimated from the theory and the value obtained from the experiment was not always this good (as illustrated in Table 1) but in general the agreement was within 0.2 v.

Table 1 also contains number-density results obtained in the freestream flow using both the microwave interferometer and voltage-swept probes. For the CO plasma, the probe-determined electron densities were in good agreement with each other (ion and electron current) but in two of the three cases they were about 30 to 40% less than the microwave results. This same general trend was reported in Ref. 12 but the reason for the disagreement is unknown. Probe and microwave-interferometer results for a nitrogen plasma are also reported in Table 1. With the exception of the final entry in the table, the number densities determined from the ion- and electron-current portions of the characteristic are in good agreement. The cause of the disagreement in the final run is not known. In all cases, the number densities determined from the ion current were in good agreement with the microwave-interferometer measurements.

Figures 5 and 6 present a comparison of the experimentally determined current-voltage characteristic with the theoretical characteristic of Laframboise. The data used here are the same as those given in Figs. 3 and 4. To obtain the comparison, the plasma potential (V_∞) was assumed to be 0.5 v consistent with

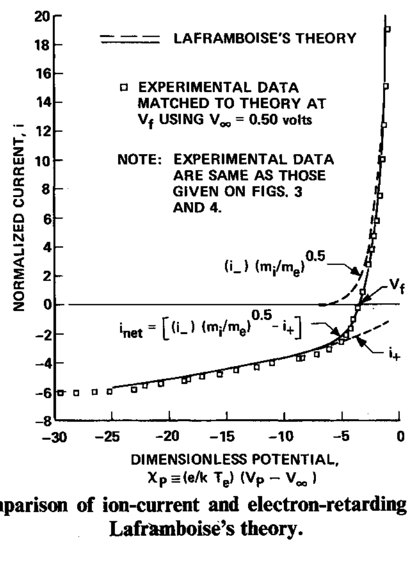

Fig. 5 Comparison of ion-current and electron-retarding regions with Laframboise's theory.

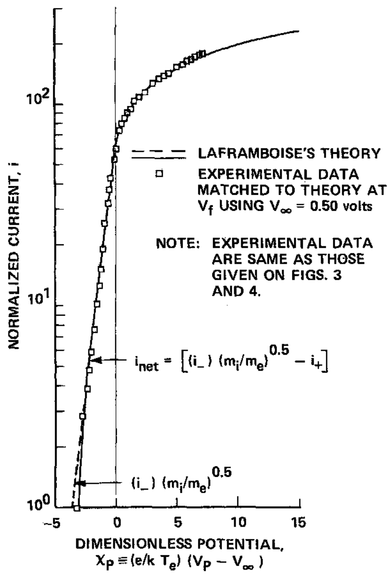

Fig. 6 Comparison of electron-retarding and electron-current regions with Laframboise's theory.

Fig. 4 and the experimental characteristic was matched to the theory at the floating potential. The ion-current and a portion of the electron-retarding regions are compared on Fig. 5. The experimental data in these regions are shown to be in good agreement with the theoretical net current. On Fig. 6, the electron-retarding and electron-current regions of the measured current-voltage characteristic are compared with Laframboise's theoretical result. The agreement between experiment and theory is also good in these regions. This comparison demonstrates that the current-voltage characteristic described by Laframboise's theory can be reproduced experimentally with thin-wire probes in a high-velocity, short-duration flow.

Conclusions

The free-molecular flow theory of Laframboise has been used to obtain values of electron temperature and electron density from Langmuir-probe current-voltage characteristics obtained in carbon monoxide and nitrogen plasmas. Measurements were obtained in the boundary-layer flow over a sharp flat plate and in the freestream of an expanding nozzle flow. Good agreement was obtained between the electron densities determined from the ion- and electron-current portions of the probe characteristics. The experimental and theoretical characteristics were found to be in good agreement. In addition, the value of $(V_\infty - V_f)$ determined from the experimental data using the tangent intercept method was found to be in reasonably good agreement with an estimated value based on the theory. In cases where an independent measurement of the electron density using microwave techniques was possible, this measurement and the probe results were found to be in reasonably good agreement.

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